NAG Fortran Library Routine Document

G01GEF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

G01GEF returns the probability associated with the lower tail of the non-central beta distribution, via the routine name.

2 Specification

```
real FUNCTION G01GEF(X, A, B, RLAMDA, TOL, MAXIT, IFAIL)
INTEGER              MAXIT, IFAIL
real                 X, A, B, RLAMDA, TOL
```

3 Description

The lower tail probability for the non-central beta distribution with parameters a and b and non-centrality parameter λ, $P(\mathrm{B} \leq \beta : a, b; \lambda)$, is defined by

$$P(B \leq \beta : a, b; \lambda) = \sum_{j=0}^{\infty} e^{-\lambda/2} \frac{(\lambda/2)}{j!} P(B \leq \beta : a, b; 0), \tag{1}$$

where

$$P(B \leq \beta : a, b; 0) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} \int_0^{\beta} B^{a-1}(1-B)^{b-1} dB,$$

which is the central beta probability function or incomplete beta function.

Recurrence relationships given in Abramowitz and Stegun (1972) are used to compute the values of $P(\mathrm{B} \leq \beta : a, b; 0)$ for each step of the summation (1).

The algorithm is discussed in Lenth (1987).

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

Lenth R V (1987) Algorithm AS226: Computing noncentral beta probabilities *Appl. Statist.* **36** 241–244

5 Parameters

1: X – ***real*** *Input*

On entry: the deviate, β, from the beta distribution, for which the probability $P(\mathrm{B} \leq \beta : a, b; \lambda)$ is to be found.

Constraint: $0.0 \leq \mathrm{X} \leq 1.0$.

2: A – ***real*** *Input*

On entry: the first parameter, a, of the required beta distribution.

Constraint: $0.0 < \mathrm{A} \leq 10^6$.

3: B – ***real*** *Input*

On entry: the second parameter, b, of the required beta distribution.

Constraint: $0.0 < \mathrm{B} \leq 10^6$.

4: RLAMDA – ***real*** *Input*

On entry: the non-centrality parameter, λ, of the required beta distribution.

Constraint: $0.0 \leq \mathrm{RLAMDA} \leq -2.0\log(U)$, where U is the safe range parameter as defined by X02AMF.

5: TOL – ***real*** *Input*

On entry: the relative accuracy required by the user in the results. If G01GEF is entered with TOL greater than or equal to 1.0 or less than 10 times the ***machine precision*** (see X02AJF), then the value of 10 times ***machine precision*** is used instead.

See Section 7 for the relationship between TOL and MAXIT.

6: MAXIT – INTEGER *Input*

On entry: the maximum number of iterations that the algorithm should use.

See Section 7 for suggestions as to suitable values for MAXIT for different values of the parameters.

Suggested value: 500.

Constraint: $\mathrm{MAXIT} \geq 1$.

7: IFAIL – INTEGER *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1. Users who are unfamiliar with this parameter should refer to Chapter P01 for details.

On exit: $\mathrm{IFAIL} = 0$ unless the routine detects an error (see Section 6).

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, because for this routine the values of the output parameters may be useful even if $\mathrm{IFAIL} \neq 0$ on exit, the recommended value is -1. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

6 Error Indicators and Warnings

If on entry $\mathrm{IFAIL} = 0$ or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

$\mathrm{IFAIL} = 1$

On entry, $\mathrm{A} \leq 0.0$,
or $\mathrm{A} > 10^6$,
or $\mathrm{B} \leq 0.0$,
or $\mathrm{B} > 10^6$,
or $\mathrm{RLAMDA} < 0.0$,
or $\mathrm{RLAMDA} > -2.0\log(U)$, where U = safe range parameter as defined by X02AMF,
or $\mathrm{X} < 0.0$,
or $\mathrm{X} > 1.0$,
or $\mathrm{MAXIT} < 1$.

If on exit $\mathrm{IFAIL} = 1$ then G01GEF returns zero.

IFAIL $= 2$

The solution has failed to converge in MAXIT iterations. The user should try a larger value of MAXIT or TOL. The returned value will be an approximation to the correct value.

IFAIL $= 3$

The probability is too close to 0.0 or 1.0 for the algorithm to be able to calculate the required probability. G01GEF will return 0.0 or 1.0 as appropriate, this should be a reasonable approximation.

IFAIL $= 4$

The required accuracy was not achieved when calculating the initial value of $P(\mathrm{B} \leq \beta : a, b; \lambda)$. The user should try a larger value of TOL. The returned value will be an approximation to the correct value.

7 Accuracy

Convergence is theoretically guaranteed whenever $\mathrm{P}(Y > \mathrm{MAXIT}) \leq \mathrm{TOL}$ where Y has a Poisson distribution with mean $\lambda/2$. Excessive round-off errors are possible when the number of iterations used is high and TOL is close to ***machine precision***. See Lenth (1987) for further comments on the error bound.

8 Further Comments

The central beta probabilities can be obtained by setting RLAMDA $= 0.0$.

9 Example

Values for several beta distributions are read, and the lower tail probabilities calculated and printed, until the end of data is reached.

9.1 Program Text

Note: the listing of the example program presented below uses ***bold italicised*** terms to denote precision-dependent details. Please read the Users' Note for your implementation to check the interpretation of these terms. As explained in the Essential Introduction to this manual, the results produced may not be identical for all implementations.

```
*     G01GEF Example Program Text
*     Mark 14 Release.  NAG Copyright 1989.
*     .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
*     .. Local Scalars ..
      real             A, B, PROB, RLAMDA, TOL, X
      INTEGER          IFAIL, MAXIT
*     .. External Functions ..
      real             G01GEF
      EXTERNAL         G01GEF
*     .. Executable Statements ..
      WRITE (NOUT,*) 'G01GEF Example Program Results'
*     Skip heading in data file
      READ (NIN,*)
      WRITE (NOUT,*)
      WRITE (NOUT,*) '     X       A       B    RLAMDA   PROB'
      WRITE (NOUT,*)
      TOL = 0.5e-5
      MAXIT = 50
   20 READ (NIN,*,END=40) X, A, B, RLAMDA
      IFAIL = -1
*
      PROB = G01GEF(X,A,B,RLAMDA,TOL,MAXIT,IFAIL)
*
      IF (IFAIL.EQ.0) THEN
         WRITE (NOUT,99999) X, A, B, RLAMDA, PROB
```

```
      ELSE
         WRITE (NOUT,99999) X, A, B, RLAMDA, PROB, ' NOTE: IFAIL = ',
     +     IFAIL
      END IF
      GO TO 20
   40 STOP
*
99999 FORMAT (1X,4F8.3,F8.4,A,I1)
      END
```

9.2 Program Data

```
G01GEF Example Program Data
 0.25  1.0  2.0  1.0         :X A B RLAMDA
 0.75  1.5  1.5  0.5         :X A B RLAMDA
 0.5   2.0  1.0  0.0         :X A B RLAMDA
```

9.3 Program Results

```
 G01GEF Example Program Results

      X       A       B    RLAMDA   PROB

    0.250   1.000   2.000   1.000  0.3168
    0.750   1.500   1.500   0.500  0.7705
    0.500   2.000   1.000   0.000  0.2500
```